UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Exam – 2019

Roll No. ....

Subject: Mathematics **Paper: I (Advanced Analysis)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION 1		
Q1	a) Prove that the unit interval I = [0,1] is not denumerable.	10
	b) Prove that $c^{\aleph_0} = c$	10
Q2	a) For cardinal number α, β and γ, prove that (i) $\alpha\,(\beta + \gamma) = \alpha\beta + \alpha\gamma$ (ii) If $\alpha \leq \beta$, then $\alpha\gamma \leq \beta\gamma$	10
	b) If A and B are similar well-ordered set then show that there exists only one similarity mapping of A onto B.	10
Q3	a) Define inequality relations between ordinal numbers and for and ordinal number λ, show that $\lambda + 1$ is the immediate successor of λ.	10
	b) If $S(A)$, the collection of all initial segments of elements in a well-ordered set A is ordered by set inclusion, then show that A is similar to $S(A)$.	10
Q4	a) Show that the axiom of choice is equivalent to Zermelo's postulate.	10
	b) Show that for the set D_m consisting of all positive divisors of integer m ordered by divisibility, the $sup(a,b)$ and $inf(a,b)$ exist for any pair $a, b \in D_m$.	10

P.T.O.

SECTION 2

Q5	a) Prove that Lebesgue outer measure is a translation invariant.	10
	b) Let $\{E_n\}$ be a decreasing sequence of measurable sets and $m(E_1) < \infty$, then show that $m(\bigcap_{i=1}^{\infty} E_i) = m(\bigcap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} m(E_n)$.	10
Q6	a) Let A be Lebesgue measurable set with $m(A) > 0$. Then show that there exists $E \subset A$ such that E is not Lebesgue measurable.	10
	b) Define L^p-Spaces. Prove that $f(x) = \frac{1}{\sqrt[3]{x}}$ belongs to $L^1[0,8]$ but does not belong to $L^3[0,8]$	10
Q7	a) If $\{f_n\}$ is a sequence of extended real- valued measurable function with same domain, then show that $\text{Inf}_{n\in\mathbb{N}} f_i$ is measurable for each n.	10
	b) Let f be an extended real-valued function defined on Borel set D. Then show that f is Borel function if and only if for each open set $V \in \overline{\mathbb{R}}, f^{-1}(V)$ is a Borel set.	10
Q8	a) Let f be bounded function defined on [a,b] which is Rieman integrable. Then show that f is measurable function and $R \int_a^b f = \int_a^b f$	10
	b) Evaluate Lebesgue integral of $f(x) = \frac{1}{x^2+1}$ on [-1,1]	10
Q9	a) State and prove Dominated Convergence Theorem.	10
	b) If f and g are non-negative measurable functions on E, then show that $\int_E f + g = \int_E f + \int_E g$	10

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Subject: Mathematics **Paper: II (Methods of Mathematical Physics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

Q. No		Marks
	Section-I	
1(a)	Use Lagrange's method to find the complete integral for the first order linear PDE $xz_x + yz_y = z.$	**10**
1(b)	Find the integral surface of the quasilinear PDE $(y^2 - z^2)z_x - xyz_y = xz$ containing the curve $x = y = z,\ x > 0.$	**10**
2(a)	Show that $$\beta F(\alpha; \beta; x) = \beta F(\alpha - 1; \beta; x) + xF(\alpha; \beta + 1; x)$$	**10**
2(b)	Find the general solution of the inhomogeneous linear second order PDE $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}.$	**10**
3(a)	Use the method of Frobenius to obtain two linearly independent series solutions about the regular singular point $x_0 = 0$ of the DE $$xy'' + (1 - x)y' - y = 0.$$	**10**
3(b)	Find the general solution of the ODE $$y'' + 2xy = 0$$ with $x_0 = 0$ as the center of expansion.	**10**
4(a)	Determine the eigenvalues and eigenfunctions of the system $y'' + \lambda y = 0$ with the boundary conditions $y(0) = y(\pi),\ \ y'(0) = 2y'(\pi).$	**10**
4(b)	Find the steady state solution of the equation $$\frac{\partial u}{\partial t} = \frac{1}{K}\frac{u^2 u}{\partial x^2} + p(x), \quad 0 < x < a, \quad t > 0$$ subject to the conditions $$u(0,t) = u_1, \quad u(a,t) = u_2, \quad t > 0, \quad u(x,0) = 0, \quad 0 < x < a.$$	**10**
5(a)	Derive the one dimensional heat equation through conduction of heat in a cylinder of finite length.	**10**
5(b)	Show that $$J^2{}_{1/2}(x) + J^2{}_{-1/2}(x) = \frac{2}{\pi x}.$$	**10**

P.T.O.

Section-II

6(a)	State and prove fundamental theorem of calculus of variation.	10
6(b)	Find the Green's function for the boundary value problem $$u'' + \lambda u = 0, \quad u(0) = u(1), \quad u'(0) = u'(1).$$	10
7(a)	Find the Fourier sine transform of the function $$f(x) = xe^{-ax}.$$	10
7(b)	Find the extremal of the function $$\int_0^{\pi/2} (y'^2 + z'^2 + 2yz)dx$$ where the boundary conditions are $$y(0) = 0, \quad y(\pi / 2) = 1, \quad z(0) = 0, \quad z(\pi / 2) = -1.$$	10
8(a)	Solve by using Fourier transform method $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \quad u(x,0) = p(x), \quad u_t(x,0) = q(x) \text{ and } u_0 > 0$$ and $u, u_x \to 0$ as $x \to \pm\infty$.	10
8(b)	If $f(x)$ is piecewise smooth and absolutely integrable function then $\lim_{\lvert k\rvert \to \infty} F(k) = 0$.	10
9(a)	Solve the DE by Laplace transform method $$y''(t) + ty'(t) - y(t) = 0, \quad y(0) = 0, \quad y'(0) = 1.$$	10
9(b)	Solve the problem by using Laplace transform method $u_{tt}(x,t) = a^2 u_{xx}(x,t), \quad t > 0, \quad x > 0, \quad u(x,0) = u_t(x,0) = 0,$ $u(0,t) = f(t)$ and $\lim_{x\to\infty} u(x,t) = 0$.	10

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Subject: Mathematics **Paper: III (Numerical Analysis)** **Time: 3 Hrs. Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

Section I

Q1.

a) State and prove Newton Raphson method to find an approximate root of the non-linear equation $f(x) = 0$.

b) Prove that Newton Raphson method is a quadratically convergent method.

c) Write an algorithm for Newton Raphson method to find an approximate root of the non-linear equation $f(x) = 0$.

(05+07+08)

Q2.

a) Find the root correct to three decimal places between 0 and 1 of the equation $1 + cost - 4t = 0$, using a numerical technique.

b) Apply Runge Kutta method of order two on $\frac{du}{dx} = Sin(xu)$, $u_0 = 0.1$, $h = 0.1$ to find $u(0.3)$.

(10+10)

Q3.

a) Write an algorithm for Heun's method for the solution of initial value problem.

b) Solve the following linear system by LU decomposition using Doolittle's method

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 2 & 2 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix}$$

(10+10)

Q4.

Solve $\frac{dy}{dx} = x^2 + y^2 + 2$ using Milne's method for $x = 0.4$. The values for $x = 0.1, 0.2,$ 0.3 should be obtained by Taylor's series method of order two.

(20)

Q5.

a) Use a numerical technique, find five approximations to the following system of equations:

$$11x + 2y + z = 15, \quad x + 10y + 2z = 16, \quad 2x + 3y - 8z = 1$$

b) Define dominant eigenvalue and dominant eigenvectors. Use the Rayleigh quotient to compute the eigenvalue λ of A corresponding to the eigenvector x.

$$A = \begin{pmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

(10+10)

P.T.O.

Section II

Q6.

a) Write an algorithm for Weddle's Rule to approximate the integral of $f(x)$ over the interval $[a, b]$ using n subintervals.

b) Using the following data, apply Lagrange's formula to find an approximate polynomial and $f(2)$.

x	-2	0	3	4
$f(x)$	25	1	-20	-23

(10+10)

Q7.

Derive 9 points Simpson's Rule. Apply 9 points Simpson's Rule to approximate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \tan(\frac{\pi x}{2}) dx$. Also find the absolute error.

(20)

Q8.

a) From the values in the table given below, find the value of $Sec 31^o$ using numerical differentiation

θ^o	31	32	33	34
$tan\theta$	0.6008	0.6249	0.6494	0.6745

b) Find a Newton polynomial matching the following data points:

$$(-2, -6), \quad (-1, 0), \quad (1, 0), \quad (2, 6), \quad (4, 60)$$

(10+10)

Q9.

a) Define difference operators. Prove that $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

b) Define difference equation. Solve:

$$y_{t+2} - 9y_{t+1} + 20y_t = 4^t(t^2 + 1)$$

(10 + 10)

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Roll No. ....

Subject: Mathematics **Paper: IV-VI (Opt. i) [Mathematical Statistics]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section. All questions carry equal marks.

		SECTION-I	Marks
Q.1	(a)	(i) Let A and B be two not-mutually exclusive events. If C is any other non-empty given event, then prove the following conditional probability, $$P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$$ (ii) If A and B are mutually exclusive and $P(C) \neq 0$, then show that the above conditional probability can be reduced as, $$P(A \cup B/C) = P(A/C) + P(B/C)$$	(10)
	(b)	What is the probability that a positive integer selected at random from first 200 positive integers is divisible by either 8 or 12 or 14?	(10)
Q.2	(a)	Show that for a very large value of N, the Hypergeometric distribution tends to the Binomial distribution.	(10)
	(b)	What is the probability that a fifth six will first appear on the eleventh roll? How many rolls should we expect to obtain seven sixes.	(10)
Q.3	(a)	Prove that if X and Y are independent Gamma variates, with parameters l and m respectively, then $\frac{X}{X+Y}$ is a $\beta_1(l, m)$ variate.	(10)
	(b)	Find the mean and the variance of a random variable Z, where Z assumes the values of absolute differences of the upper faces of two dices one of which is green and the other is red. Further, if green is labelled by X and red is labelled by Y, then find (i) $P(X + Y > 13)$ (ii) $P(X + Y < 8)$	(10)
Q.4	(a)	Prove that the Normal distribution is symmetrical. That is, mean, mode and median are equal.	(10)
	(b)	In a Statistics class test, the marks obtained were normally distributed with a mean of 66 and a standard deviation of 4. What proportion of the class would be expected to score between 58 and 72 points? How many scores will it take for 77% marks of the overall class?	(10)

P.T.O.

SECTION-II

Q.5	**(a)**	Define partial correlation coefficients and establish if $r_{12.3} = 1$, then $r_{13.2} = 1 = r_{32.1}$	(10)
	(b)	List all the six partial regression coefficients and write their formulas. Prove that the standard equation of the regression plane passing through the origin (as the mean point) can be written as $\frac{x_1}{s_1} = (\frac{r_{12} - r_{13}r_{23}}{1-r_{23}^2})\frac{x_2}{s_2} + (\frac{r_{13} - r_{12}r_{23}}{1-r_{23}^2})\frac{x_3}{s_3}$.	(10)
Q.6	**(a)**	Given the joint density $f(x,y) = \begin{cases} \alpha x + \alpha y & for \quad 0 < x < 1 \; and \; 0 < y < 1 \\ 0 & elsewhere \end{cases}$ Find α, $\mu_{Y/x}$ and $\mu_{X/y}$.	(10)
	(b)	Find the moment generating function of random variable X, whose moments about origin are given by $\mu_r' = 2^r(r+1)!$	(10)
Q.7	**(a)**	If the joint probability density of X and Y is given by $f(x,y) = \begin{cases} 16xy, & for \quad 0 < x < 2 \; and \; 0 < y < 2 \\ 0 & elsewhere \end{cases}$ Find the probability density function of $Y_1 = XY, \;\; Y_2 = X + Y$	(10)
	(b)	Find the moment generating function of Uniform distribution and discuss its coefficeients of skewness and kurtosis.	(10)
Q.8	**(a)**	Establish the probability function for χ^2-distribution.	(10)
	(b)	Establish the probability function for F-distribution. Also find its Mode.	(10)
Q.9	**(a)**	Find the coefficient of skewness and and kurtosis for the χ^2-distribution.	(10)
	(b)	Prove that all even order moments about origin of t-distribution with n degree of freedom is given by $$\frac{n^r\Gamma(r+\frac{1}{2})\Gamma(\frac{n}{2}-r)}{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})}$$	(10)

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M.A./M.Sc. Part – II Annual Exam – 2019

Roll No. ....

Subject: Mathematics **Paper: IV-VI (Opt. ii) [Computer Applications]** **Time: 3 Hrs.** **Marks: 50**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

Section I

Q1. Write a program to find the smallest element in an array of twenty elements using Function Subprogram. (10)

Q2. **a)** Write a program to find the complex roots of a quadratic equation.

b) Write a FORTRAN expression corresponding to the following Mathematical expression:

$$\frac{\sqrt{x^2} + Sinx + e^{x^3+e^x}}{|x - \sin x| + \log x^{2x}}$$

(05+05)

Q3. Define loops, arrays and Format in FORTRAN Programming. (10)

Q4. Write a program to find the area of a triangle when two sides and angle between them are given, using Subroutine Subprogram.

(10)

Section II

Q5. Write a program to find the value of the following integral $\int_4^{10} \frac{1}{x}$ using Trapezoidal Rule. Compare the approximate value with the exact value; find the absolute error and relative error. (10)

Q6. Write a program to solve the following system of equations using Jacobi iterative method:

$$8x - 3y + 2z = 20, \quad 4x + 11y - z = 33, \quad 6x + 3y + 12z = 35$$ (10)

Q7. From the following table write a program to find $f(0.5)$ using Lagrange interpolating formula.

P.T.O.

Table

x	-1	0	2	3
$f(x)$	-1	0	8	27

(10)

Q8. Write a program to solve the following system of differential equation by Runge Kutta method of order two:

$$\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1, \text{ over the interval } [0,2].$$

(10)

Q9. Write the Mathematica statements for the following:

1. Evaluate $\frac{d}{dx}\left(\log_3^{x^2+4x}\right)$.
2. Evaluate $\int_0^1 \sec x dx$.
3. Plot the graph of $\tan x$, $Sec(x)$, $-\frac{\pi}{4} < x < \frac{+\pi}{4}$.
4. Find the conjugate of $z = x + iy$
5. Find the sum of the series $2 + 4 + 6 + 8 + \cdots + 2^n$.

(10)

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Subject: Mathematics **Paper: IV-VI (Opt. iii) [Group Theory]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

		SECTION-I	Marks
Q.1	(a)	Prove that a finite group whose order is divisible by a prime p contains a Sylow p-subgroup.	(10)
	(b)	Let A and B be two cyclic groups of order n and m, respectively. Then show that direct product of A and B is cyclic group of order nm if and only if $\gcd(n,m)=1$. Also, find the number of elements of order 3 in a group $G = Z_9 \oplus Z_3$.	(10)
Q.2	(a)	Show that the number k of Sylow p-subgroups of a finite group G is congruent to 1 mod p.	(10)
	(b)	Show that for any prime divisor p of the order n of a goup G, G has a unique Sylow p-subgroup H if and only if H is normal in G.	(10)
Q.3	(a)	Show that the center of a group G is a characteristic subgroup of G.	(10)
	(b)	What is meant by the holomorph of a group G? Find the holomorph of the group $G = \langle a, b: a^3 = b^2 = (ab)^2 = 1 \rangle$.	(10)
Q.4	(a)	Let H be characteristic subgroup of a normal subgroup of a group G. Then prove that H is normal subgroup of G.	(8)
	(b)	State and prove Orbit Stabalizer Theorem.	(8)
	(c)	Discuss the simplicity of A_4.	(4)

P.T.O.

SECTION-II

Q.5	(a)	State and prove Zassenhaus Butterfly Lemma.	(10)
	(b)	Define the normal series of a group G. What is a subnormal subgroup of a group? Give an example of a group in which a subnormal subgroup is not necessarily a normal subgroup of the group?	(10)
Q.6	(a)	Show that a group G is solvable if and only if it has a normal series with abelian factors.	(10)
	(b)	Define a nilpotent group and its nilpotency class, with illustration of an example. Is every solvable group also nilpotent and vice versa? Justify your answer.	(10)
Q.7	(a)	Construct lower and upper central series of dihedral group D_8 of order eight .	(6)
	(b)	Show that every term $\xi_i(G)$ in upper central series of a group G is characteristic subgroup of G.	(6)
	(c)	Let G be a nilpotent group and H be its proper subgroup. Then show that H is properly contained in its normalizer.	(8)
Q.8	(a)	Define the Frattini subgroup of a group. Write the Frattini subgroups of A_4 and of the group Q_8 of quaternions.	(10)
	(b)	Define partial complement of a subgroup and prove that a normal subgroup H of a group G is contained in Frattini subgroup of G if and only if H has no partial complement in G.	(10)
Q.9	(a)	Define special linear group and projective linear group. Prove that general linear group is not simple.	(10)
	(b)	Write a note on the following: i) General Linear Groups ii) Representation of a group	(10)

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Roll No. ....

Subject: Mathematics **Paper: (IV-VI) (Opt. iv) [Rings & Modules]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section I

Q. 1. a) Let R be an integral domain, R is Unique Factorization Domain if and only if R is a factorization domain and every irreducible element is prime.

b) Prove that

$R = \{a + b\sqrt{-5} : a, b \in Z\}$ is not a unique Factorization Domain. 10+10

Q. 2. a) Let R be an integral domain, $p \in R\backslash\{0\}$. Then p is prime if and only if $R/_{pR}$ is an integral domain.

b) By giving an example, justify that an irreducible element in $\mathbb{Z}[\sqrt{-3}]$ may not be prime. 10+10

Q. 3. a) Prove that $\mathbb{Z}[i]$, the ring of Gaussian integers, is Euclidean domain.

b) If R is an integral domain, show that

(i) s|t if and only if $tR \subseteq sR$

(ii) u is a unit of R if and only if $uR = R$

(iii) the set of all units of R is an abelian group with respect to multiplication. 10+10

Q. 4. a) Let $f(x) = x^3 - 2 \in \mathbb{Q}[x]$. Show that the splitting field of f(x) over $\mathbb{Q}$ is $K = \mathbb{Q}(\sqrt[3]{2}, \sqrt{3}i)$ and find $[K: \mathbb{Q}]$.

b) If L is finite extension of K and K is a finite extension of F, then prove that L is finite extension of F and $[L: F] = [L: K][K: F]$. 10+10

Q. 5. a) Let D be an integral domain, let F be a field such that $F \subseteq D$. Suppose unity 1 of F is also unity of D. Then D can be regarded as a vector space over F. Show that D is a field if $[D: F] =$ finite.

b) Find the splitting fields of the following polynomials over $\mathbb{Q}$.

i. $x^4 - 1$
ii. $x^4 - x^2 - 2$

10+10

P.T.O.